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Size and Power of a Sometimes Pool Test Procedure in a Mixed Anova Model Using Two Preliminary Tests of Significance

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Summary

The paper presents an alternative test procedure involving two preliminary tests of significance (PTS) for testing the treatment differences in the analysis of variance (ANOVA) mixed model. The size and power of the sometimes pool test procedure (SPT) have been numerically obtained and compared with those of (i) never pool test (NPT) procedure and (ii) sometimes pool test procedure due to Ali and Srivastava [2] [3].

Key words : ANOVA, Sometimes pool test, Never pool test, NID.

Introduction

Ali and Srivastava [3] considered the following conditionally specified mixed ANOVA model corresponding to a split plot in time experiment which has frequent use in forage crops (Steel and Torrie [13]),

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \delta_{ij} + \tau_k + (\alpha\tau)_{ik} + (\beta\tau)_{jk} + \epsilon_{ijk} \tag{1.1}$$

where, Y_{ijk}= Yield on the k^{th} cutting of the j^{th} variety in the i^{th} block, i = 1, 2,. . .,r; j=1, 2,. . . , s; k= 1, 2, . . ., t; μ is the true mean effect, α_i is the random block effect and β_j, τ_k are the fixed effects of varieties and cuttings respectively, δ_{ij} is the true effect of the whole experimental unit with the j^{th} variety in the i^{th} block, $(\alpha\tau)_{ik}$ is the interaction effect between j^{th} block and k^{th} cutting, $(\beta\tau)_{jk}$ is the interaction effect between j^{th} variety and k^{th} cutting, while ϵ_{ijk} being the true effect of the whole experimental unit subjected to the k^{th} cutting of j^{th} variety in the i^{th} block i.e. an effect of error term. The distributions of various parameters and the constrains of the above model are as follows:

$$\epsilon_{ijk} \sim NID(0, \sigma^2_\epsilon),\ \alpha_i \sim NID(0, \sigma^2_\alpha),\ \delta_{ij} \sim NID(0, \sigma^2_\delta), (\alpha\tau)_{ik} \sim NID(0, \sigma^2_{\alpha\tau});$$

$$\sum_{j=1}^{s} \beta_j = 0, \sum_{k=1}^{t} \tau_k = 0, \sum_{k=1}^{t} (\alpha\tau)_{ik} = 0, \sum_{i=1}^{r} (\alpha\tau)_{ik} \neq 0,$$

$$\sum_{j=1}^{s} (\beta\tau)_{jk} = 0, \sum_{k=1}^{t} (\beta\tau)_{jk} = 0.$$

The cuttings effect, i.e. τ_k, is of main interest for which the abridged ANOVA table is as follows.

Table 1. Mixed model abridged ANOVA for a split-plot in time experiment

Source of variation	Degrees of freedom	Mean squares	Expected mean squares
Treatments (Cuttings)	$n_4 = t - 1$	V_4	$\sigma_4^2 = \sigma_\epsilon^2 + s\,\sigma_{\alpha\tau}^2 + rs[\sigma_\tau^2]$ $= \sigma_3^2 (1 + 2\,\Omega_4/n_4)$
True Error (Cuttings × Block)	$n_3 = (t-1)(r-1)$	V_3	$\sigma_3^2 = \sigma_\epsilon^2 + s\sigma_{\alpha\tau}^2$
Doubtful Error II (Cuttings × Varieties)	$n_2 = (t-1)(s-1)$	V_2	$\sigma_2^2 = \sigma_\epsilon^2 + r[\sigma_{\beta\tau}^2]$ $= \sigma_1^2 (1 + 2\,\Omega_2/n_2)$
Doubtful Error I (Cuttings × Variety × Block)	$n_1 = (t-1)(s-1)(r-1)$	V_1	$\sigma_1^2 = \sigma_\epsilon^2$

In Table 1 Ω_2 and Ω_4 are the non- centrality parameters. It may be noted that the model (1.1) applies to any three way cross classification layout where any two factors may be fixed effects and the third being random.

To describe the testing situation, we have the doubtful condition that $(\alpha\tau)_{ik}$ and/or $(\beta\tau)_{jk}$ may equal to zero, i.e. $\sigma_{\alpha\tau}^2$ and/or $\sigma_{\beta\tau}^2$ may equal to zero (see Table 1). In other words, σ_3^2 and/or $\sigma_2^2 \geq \sigma_1^2$. The main hypothesis of no treatment differences is $H_0 : \sigma_4^2 = \sigma_3^2$ (i.e. $\Omega_4 = 0$) against the alternative hypothesis H_1: $\sigma_4^2 > \sigma_3^2$ (i.e. $\Omega_4 > 0$) where σ_4^2 and σ_3^2 are the true treatment and error variances respectively. When $\sigma_3^2 \neq \sigma_2^2 \neq \sigma_1^2$, the usual never pool test of H_0 is $F = V_4/V_3$. However, in the present context the doubtful condition as stated above exists.

To resolve these uncertainties Ali and Srivastava considered the preliminary tests $H_{01}: \sigma_3^2 = \sigma_1^2$ vs $H_{11}: \sigma_3^2 > \sigma_1^2$ and $H_{02}: \sigma_2^2 = \sigma_1^2$ (i.e. $\Omega_2 = 0$) vs $H_{12}: \sigma_2^2 > \sigma_1^2$ (i.e. $\Omega_2 > 0$) in succession on the outcomes of which they based their final tests.

Since the final tests depend on the order in which the two preliminary tests are performed, and further since σ_3^2 is already the true error variance it seems more appropriate to first test the equality of doubtful error variance σ_2^2 with the usual error variance σ_1^2, which is also the doubtful error in the present context. Thus, first carry out the preliminary tests $H_{01}: \sigma_2^2 = \sigma_1^2$ (i.e. $\Omega_2 = 0$) vs $H_{11}: \sigma_2^2 > \sigma_1^2$ (i.e. $\Omega_2 > 0$) and $H_{02}: \sigma_3^2 = \sigma_1^2$ vs $H_{12}: \sigma_3^2 > \sigma_1^2$ in succession before testing the main hypothesis $H_0: \sigma_4^2 = \sigma_3^2$ (i.e. $\Omega_4 = 0$) vs $H_1: \sigma_4^2 > \sigma_3^2$ (i.e. $\Omega_4 > 0$). The mathematical statement of the resulting alternative test procedure is thus as follows :

Reject $H_0: \sigma_4^2 = \sigma_3^2$(i.e. $\Omega_4 = 0$) vs $\{H_1: \sigma_4^2 > \sigma_3^2\}$ (i.e.$\Omega_4 > 0$) if any one of the following four mutually excursive events E_1, E_2, E_3 or E_4 occurs :

$$E_1: \{V_2/V_1 \geq F(n_2, n_1; \alpha_1), V_3/V_1 \geq F(n_3, n_1; \alpha_2), V_4/V_3 \geq F(n_4, n_3; \alpha_3)\}$$

$$E_2: \{V_2/V_1 \geq F(n_2, n_1; \alpha_1), V_3/V_1 < F(n_3, n_1; \alpha_2), V_4/v_{13} \geq F(n_4, n_{13}; \alpha_4)\}$$

$$E_3: \{V_2/V_1 > F(n_2, n_1; \alpha_1), V_3/V_{12} \geq F(n_3, n_{12}; \alpha_5), V_4/V_3 \geq F(n_4, n_3; \alpha_3)\}$$

$$E_4: \{V_2/V_1 < F(n_2, n_1; \alpha_1), V_3/V_{12} < F(n_3, n_1; \alpha_5), V_4/V_{123} \geq F(n_4, n_{123}; \alpha_6)\} \tag{1.2}$$

where $V_{12} = (n_1 V_1 + n_2 V_2)/(n_1 + n_2)$, $V_{13} = (n_1 V_1 + n_3 V_3)/(n_1 + n_3)$, $V_{123} = (n_1 V_1 + n_2 V_2 + n_3 V_3)/(n_1 + n_2 + n_3)$ are the different pooled mean squares with respective degrees of freedom $n_{12} = n_1 + n_2$, $n_{13} = n_1 + n_3$, $n_{123} = n_1 + n_2 + n_3$ and $F(n_i, n_j; \alpha_k)$ is the upper 100 α_k% point of the central F-distribution with (n_i, n_j) degrees of freedom.

2. *Derivation of the Power Function for the Proposed Test*

In order to find the power function, obtain first the joint density function of V_i, i = 1, 2, 3, 4, namely

$$f(V_1, V_2, V_3, V_4) = A\, V_1^{\frac{1}{2}n_1 - 1}\, V_2^{\frac{1}{2}v_2 - 1}\, V_3^{\frac{1}{2}n_3 - 1}\, V_4^{\frac{1}{2}v_4 - 1}$$
$$\exp[-\tfrac{1}{2}\{n_1 V_1/\sigma_1^2 + n_2 V_2/(\sigma_1^2 c_2) + n_3 V_3/\sigma_3^2 + n_4 V_4/(\sigma_3^2 c_4)\}] \tag{2.1a}$$

where,

$$A = \frac{(n_1/\sigma_1^2)^{\frac{1}{2}n_1} \; (n_2/(\sigma_1^2 c_2))^{\frac{1}{2}v_2} (n_3/\sigma_3^2)^{\frac{1}{2}n_3} (n_4/(\sigma_3^2 c_4))^{\frac{1}{2}v_4}}{2^{\frac{1}{2}(n_1+v_2+n_3+v_4)} \Gamma(\frac{1}{2}n_1) \Gamma(\frac{1}{2}v_2) \Gamma(\frac{1}{2}v_4)} \tag{2.1b}$$

after using the Patnaik's [9] approximation to non-central Chisquares (for V_2 and V_4, so that

$$v_i = n_i + \frac{4\Omega_i^2}{n_i + 4\Omega_i}, \; c_i = 1 + \frac{2\Omega_i}{n_i + 2\Omega_i} \tag{2.1c}$$

where we calculate Ω_i and c_i, by the iteration process, such that v_i's are always positive integers. However, during numerical evaluation of power components (Section 3), we have used only even positive integral values of v_i's for the sake of finiteness of binomial expansions.

Introducing the transformations :

$$u_1 = n_4 V_4/(n_3 V_3 c_4), u_2 = n_2 V_2/(n_2 V_1 c_2), u_3 = n_3 V_3/(n_1 V_1 \theta_{31}),$$

$$u_4 = n_1 V_1/(2\sigma_1^2))\} \tag{2.2}$$

where

$$0 \le u_1 < \infty, 0 \le u_2 < \infty, 0 \le u_3 < \infty, 0 \le u_4 < \infty; \theta_{31} = \sigma_3^2/\sigma_1^2,$$

and integrating out u_4 from (2.1a) over its range $0 \le u_4 \le \infty$, the joint density function can be rewritten as

$$f(u_1, u_2, u_3) = A_2 \frac{u_1^{\frac{1}{2}v_4 - 1} u_2^{\frac{1}{2}v_2 - 1} u_3^{\frac{1}{2}(n_3+v_4)-1}}{(1 + u_2 + u_3 + u_1 u_3)^{\frac{1}{2}(n_1+v_2+n_3+v_4)}} \tag{2.3a}$$

where,

$$A_2 = \frac{\Gamma(\frac{1}{2}(n_1 + v_2 + n_3 + v_4))}{\Gamma(\frac{1}{2}n_1)\Gamma(\frac{1}{2}v_2)\Gamma(\frac{1}{2}n_3)\Gamma(\frac{1}{2}v_4)} \tag{2.3b}$$

The power function P of the test is the simple addition of probabilities P_i assigned with events E_i.

Now, to derive P_i we express $\{V_2/V_1 \geq F(n_2, n_1; \alpha_1), V_3/V_1 \geq F(n_3, n_1; \alpha_2), V_4/V_3 \geq F(n_4, n_3; \alpha_3)\}$ in terms of u's as under :

$$\{V_2/V_1 \geq F(n_2, n_1; \alpha_1), V_3/V_1 \geq F(n_3, n_1; \alpha_2), V_4/V_3 \geq F(n_4, n_3; \alpha_3)\}$$
$$= \{u_2 \geq a, u_3 \geq b, u_1 \geq c\}$$

where,

$$a = u_1^0/c_2, u_1^0 = (n_2/n_1)\, F(n_2, n_1; \alpha_1)$$
$$b = u_2^0/\theta_{31}, u_2^0 = (n_3/n_1)\, F(n_3, n_1; \alpha_2)$$
$$c = u_3^0/c_4, u_3 = (n_4/n_3)\, F(n_4, n_3; \alpha_3) \tag{2.4}$$

Thus using (2.4) and (2.3) for the event E_1, we get the integral for P_1 as follows :

$$P_1 = A_2 \int_{u_2=a}^{\infty} \int_{u_3=b}^{\infty} \int_{u_1=c}^{\infty} f(u_1, u_2, u_3)\, du_1\, du_3\, du_2 \tag{2.5}$$

First integrate out u_1 by using the transformation :

$$z = \frac{1 + u_2 + u_3}{1 + u_2 + u_3 + u_1 u_3}$$

so that,

$$u_1 = \frac{(1 + u_2 + u_3)}{u_3} \frac{(1 - z)}{z}, \quad du_1 = -\frac{(1 + u_2 + u_3)}{u_3} \frac{dz}{z^2} \tag{2.6}$$

and get
$$P_1 = A_2 \int_{u_2=a}^{\infty} \int_{u_3=b}^{\infty} \frac{u_2^{\frac{1}{2}v_2 - 1}\, u_3^{\frac{1}{2}n_3 - 1}}{(1 + u_2 + u_3)^{\frac{1}{2}(n_1 + v_2 + n_3)}}$$
$$\left[\int_0^z z^{\frac{1}{2}(n_1 + v_2 + n_3) - 1} (1 - z)^{\frac{1}{2}v_4 - 1}\, dz\right] du_2\, du_3$$
$$\text{where } z = \frac{1 + u_2 + u_3}{1 + u_2 + (1 + c)u_3}$$

Expanding $(1 - z)^{\frac{1}{2}v_4 - 1}$ binomially and integrating term by term with respect to z, we get

$$P_1 = A_2 \sum_{i=0}^{\frac{1}{2}v_4 - 1} \frac{(-1)^i \binom{\frac{1}{2}v_4 - 1}{i}}{\frac{1}{2}(n_1 + v_2 + n_3) + i}$$

$$\int_{u_2=a}^{\infty} \int_{u_3=b}^{\infty} \frac{u_2^{\frac{1}{2}v_2 - 1} u_3^{\frac{1}{2}n_3 - 1} (1 + u_2 + u_3)^i}{(1 + u_2 + (1+c)u_3)^{\frac{1}{2}(n_1 + v_2 + n_3) + i}} du_2 \, du_3 \tag{2.7}$$

Besides the remarks on the values of v_i,s on page 4, it may be further remarked here that, in the foregoing and subsequent binomial expansions, the indices are assumed to be finite positive integers lest the expansions become infinite. During numerical evaluations of power components (Section 3) this assumption holds true.

Next u_3 is integrated out after the binomial expansion of $(1 + u_2 + u_3)^i$ in terms of u_3 and $(1 + u_2)$ using the transformation,

$$y = \frac{1 + u_2}{1 + u_2 + (1+c)u_3} \Rightarrow u_3 = \frac{(1+u_2)}{(1+c)} \frac{(1-y)}{y}, \quad du_3 = -\frac{(1+u_2)}{(1+c)} \frac{dy}{y^2} \tag{2.8}$$

This gives an expression involving the term $(1-y)^{\frac{1}{2}n_3 + j - 1}$ which is again binomially expanded and the integration is made subsequently with respect to y to give

$$P_1 = A_2 \sum_{i=0}^{\frac{1}{2}v_4 - 1} \frac{(-1)^i \binom{\frac{1}{2}v_4 - 1}{i}}{\frac{1}{2}(n_1 + v_2 + n_3 + i} \sum_{j=0}^{i} \frac{\binom{i}{j}}{(1+c)^{\frac{1}{2}n_3 + j}}$$

$$\sum_{k=0}^{2n_3 + j - 1} \frac{(-1)^k \binom{\frac{1}{2}n_3 + j - 1}{k}}{2(n_1 + v_2) + i - j + k} \int_{u_2=a}^{\infty} \frac{u_2^{\frac{1}{2}v_2 - 1} (1 + u_2)^{i-j+k}}{(1 + u_2 + (1+c)b)^{\frac{1}{2}(n_1 + v_2) + i - j + k}} du_2 \tag{2.9}$$

Finally the binomial expansion of $(1 + u_2)^{i-j+k}$ and the application of the transformation,

$$t = \frac{1 + (1+c)b}{1 + u_2 + (1+c)b} \Rightarrow u_2 = \{1 + (1+c)b\} \frac{(1-t)}{t},$$

$$du_2 = -\{1 + (1+c)b\} \frac{dt}{t^2} \tag{2.10}$$

yields P_1 as

$$P_1 = A_2 \sum_{i=0}^{\frac{1}{2}v_4-1} \frac{(-1)^i \binom{\frac{1}{2}v_4-1}{i}}{\frac{1}{2}(n_1+v_2+n_3)+i} \sum_{j=0}^{i} \frac{\binom{i}{j}}{(1+c)^{\frac{1}{2}n_3+j}}$$

$$\sum_{k=0}^{\frac{1}{2}n_3+j-1} \frac{(-1)^k \binom{\frac{1}{2}n_3+j-1}{k}}{\frac{1}{2}(n_1+v_2)+i-j+k} \sum_{l=0}^{i-j+k} \binom{i-j+k}{l} \frac{Bx_1\left(\frac{1}{2}n_1+i-j+k-1,\ \frac{1}{2}v_2+1\right)}{\{1+(1+c)b\}^{\frac{1}{2}n_1+i-j+k-1}}$$

(2.11a)

where A_2 is given by (2.3b), $B_x(m, n) = \int_0^x y^{m-1}(1-y)^{n-1}\,dy$

and $$x_1 = \frac{1+(1+c)b}{1+a+(1+c)b} \qquad (2.11b)$$

The expressions for P_2, P_3 and P_4 have been obtained in the similar manner and are as follows.

$$P_2 = A_2 \sum_{i=0}^{\frac{1}{2}v_4-1} \frac{(-1)^i \binom{\frac{1}{2}v_4-1}{i}}{\frac{1}{2}(n_1+v_2)+n_3)+i} \sum_{j=0}^{i} \frac{\binom{i}{j}}{(1+e)^{\frac{1}{2}n_3+j}}$$

$$\sum_{k=0}^{\frac{1}{2}n_3+j-1} \frac{(-1)^k \binom{\frac{1}{2}n_3+j-1}{i}}{\frac{1}{2}(n_1+v_2)+i-j+k} \sum_{l=0}^{i-j} \binom{i-j}{l} \left[\frac{Bx_{21}\left(\frac{1}{2}n_1+i-j-1,\ \frac{1}{2}v_2+1\right)}{(1+d)^{\frac{1}{2}n_1+i-j-1}}\right.$$

$$\left. -\sum_{m=0}^{k} \binom{k}{m} \frac{(1+d)^{k-m}\, Bx_{22}\left(\frac{1}{2}n_1+i-j+k-1-m,\ \frac{1}{2}v_2+1+m\right)}{\{1+d+(1+e)b\}^{\frac{1}{2}n_1+i-j+k-1-m}}\right]$$

(2.12a)

$$x_{21} = \frac{1+d}{1+d+a}, \quad x_{22} = \frac{1+d+(1+e)b}{1+d+a+(1+e)b} \qquad (2.12\text{ b})$$

$$P_3 = A_2 \sum_{i=0}^{\frac{1}{2}v_4-1} \frac{(-1)^i \binom{\frac{1}{2}v_4-1}{i}}{\frac{1}{2}(n_1+v_2+n_3)+i} \sum_{j=0}^{i} \frac{\binom{i}{j}}{(1+c)^{\frac{1}{2}n_3+j}}$$

$$\sum_{k=0}^{\frac{1}{2}n_3+i-1} \frac{(-1)^k \binom{\frac{1}{2}n_3+j-1}{k}}{\frac{1}{2}(n_1+v_2)+i-j+k} \sum_{l=0}^{i-j+k} \binom{i-j+k}{1} \frac{Bx_3\left(\frac{1}{2}v_2+1,\ \frac{1}{2}n_1+i-j+k-1\right)}{\{1+(1+c)f\}^{\frac{1}{2}n_1+i-j+k-1}\,\{1+(1+c)g\}^{\frac{1}{2}v_2+1}}$$

(2.13a)

$$x_3 = \frac{\{1+(1+c)g\}\,a}{\{1+(1+c)f\}+\{1+(1+c)g\}\,a} \tag{2.13 b}$$

$$P_4 = A_2 \sum_{i=0}^{\frac{1}{2}v_4-1} \frac{(-1)^i \binom{\frac{1}{2}v_4-1}{i}}{\frac{1}{2}(n_1+v_2+n_3)+i} \sum_{j=0}^{i} \frac{\binom{i}{j}}{(1+q)^{\frac{1}{2}n_3+j}}$$

$$\sum_{k=0}^{\frac{1}{2}n_3+j-1} \frac{(-1)^k \binom{\frac{1}{2}n_3+j-1}{k}}{\frac{1}{2}(n_1+v_2+i-j+k} \sum_{l=0}^{i-j} \binom{i-j}{l} \left[\frac{Bx_{41}(\frac{1}{2}v_2+1, \frac{1}{2}n_1+i-j-1)}{(1+h)^{\frac{1}{2}n_1+i-j-1}(1+p)^{\frac{1}{2}v_2+1}}\right.$$

$$\left.-\sum_{m=0}^{k}\binom{k}{m} \frac{(1+p)^m(1+h)^{k-m}Bx_{42}(\frac{1}{2}v_2+1+m, \frac{1}{2}n_1+i-j+k-1-m)}{\{(1+h)+(1+q)f\}^{\frac{1}{2}n_1+i-j+k-1-m}\{(1+p)+(1+q)g\}^{\frac{1}{2}v_2+1+m}}\right] \tag{2.14 a}$$

$$x_{41} = \frac{(1+p)a}{(1+h)+(1+p)a},\; x_{42} = \frac{\{(1+p)+(1+q)g\}\;a}{(1+h)+(1+q)f+\{(1+p)+(1+q)g\}\,a} \tag{2.14 b}$$

3. *Illustration and Discussion*

In order to examine the merit of the proposed test procedure the size and power of the test have been calculated for the set of the parameters considered by Ali [1] and presented the result in Table 2.

Table 2. Magnitude of Maximum size for $\Omega_2 = 0$, $\alpha_f = 0.05$

Degrees of freedom				Preliminary levels of significance (α_p)			
n_1	n_2	n_3	n_4	Ali's test* procedure		The new proposed test procedure	
				0.25	0.50	0.25	0.50
2	2	2	2	0.12345	0.07041	0.09747	0.05976
2	2	4	2	–	–	0.06782	0.05153
2	2	2	4	–	–	0.11706	0.06793
10	2	2	2	0.12703	0.07140	0.11980	0.06807
10	10	2	2	0.13700	0.07665	0.12283	0.06940
10	10	10	2	0.06931	0.05229	0.05922	0.04974
10	10	10	4	0.08074	0.05526	0.06601	0.05220
30	2	2	2	0.13046	0.07366	0.12830	0.07244
30	20	2	2	0.13741	0.07766	0.13604	0.07376

* These entries were extracted from Ali [1] for comparison.

A perusal of Table 2 indicates that for the proposed test the maximum size remains uniformly below 0.075, the tolerance limit adopted by Gupta [6], Saxena et.al. [11] and Ali [1]. Further more, change in the order of preliminary test giving rise to the new procedure always yields smaller size as compared to Ali's case.

It may be further observed that the maximum size decreases with an increase in the preliminary level of significance. This result agrees with those of Bozivich et.al [4] [5], Paull [10], Srivastava et.al. [12] and [6] Ali [1] but is in contrast with the studies made by Gupta [6], Gupta et.al [7] and Saxena et.al. [11] where an increase in size maximum is reported to be associated with the increase in the preliminary level. The difference in the results on size maximum may be explained by the difference in the models used by Gupta [6], Gupta et. al. [7] and Saxena et. al. [11]. Theirs was a three-way fixed effect model. The present one is a three-way mixed effect model. And the agreement between the present mixed effect model and the random effect models of Bozivich the present mixed effect model and the random effect models of Bozivich the present mixed effect model and the random effect models of Bozivich et. al. [4] [5], Paull [10], Srivastava et. al. [12] may be explained by an approximate technique given by Bozivich et. al. [4] [5] to reduce mixed effect models to random effect models. This shows that the mixed effect models are closer to random ones. Similar types of difference in size maximum may also be found between the random effect model studies of Bozivich et. al. [5] and Mead et.al. [8].

It may also be observed from the Table 2 that the maximum size (i) increases with the increase in n_1 for fixed values of n_2 , n_3 and n_4; (ii) increases as n_2 increases for fixed values of n_1, n_2 and n_4; (iii) decreases with the increase in n_3 for fixed values of n_1, n_2 and n_4; and (iv) increases with the increase in n_4 for fixed values of n_1, n_2 and n_3.

Based on these observations and keeping $\alpha_p = 0.50$ with a size tolerance of 0.075 we may form some satisfactory sets of degrees of freedom ensuring adequate size control for the proposed test procedure. These are shown in Table 3.

It can be seen that the situations of the mixed model under study give rise to several satisfactory sets of degrees of freedom. For example, $n_1 = 10, n_2 = 2, n_3 = 10, n_4 = 2$ is a satisfactory set of degrees of freedom which arises from the experiment with 2 varieties (s = 2), 6 blocks (r = 6) and 3 cuttings (t = 3) ensuring a size maximum less than 0.06807. Similarly, with r = 2, s = 4, t = 3 we have the satisfactory set of degrees of freedom $n_1 = 6, n_2 = 6, n_3 = 2, n_4 = 2$ through which a size maximum less than 0.06940 may be ensured.

Table 3. Satisfactory Sets of Degrees of Freedom for $\alpha_f = 0.05$

Degrees of Freedom				Upper Limit to Maximum Size
n_1	n_2	n_3	n_4	
2	2	2	2	0.05976
2	2	4	2	0.05153
≤ 2	≤ 2	≥ 2	≤ 4	0.06793
≤ 10	≤ 2	≥ 2	≤ 2	0.06807
≤ 10	≤ 10	≥ 2	≤ 2	0.06940
10	10	10	2	0.04974
10	10	10	4	0.05220
≤ 30	≤ 2	≥ 2	≤ 2	0.07244
≤ 30	≤ 20	≥ 2	≤ 2	0.07376

N.B. : The inequalities attached with n_i's in the table operate in accordance with the four observations (i), (ii), (iii) and (iv) made on n_i's described above.

Thus, while planning for the actual experiment if one does not ensure the satisfactory combinations of degrees of freedom as above, he is liable to lose control over the size maximum for the proposed test procedure and in that case the size tolerance may exceed 0.075.

For various combinations of parameters, we calculated the power of the proposed test and compared with the existing results. It was found that gain in power was almost negligible as is evident from the Table 4 (a, b).

4. *Conclusion*

In the present conditional specification of the model (1.1) it is more appropriate to first test the equality of doubtful error variance σ_2^2 with the usual error variance σ_1^2, (which is also the doubtful error in the present context) because the resulting test procedure always has greater size control and has equal power as compared to Ali's case. It may be suggested here that more studies of this nature should be undertaken in case of other conditionally specified models to confirm the order to preliminary test giving rise to tests with greater size control and having equal or more power.

Table 4. The Power Comparison of the Sometimes Pool Test Procedure Involving Two Preliminary Tests With That of Ali [1]* (From Table 4(a) & 4(b))

Table 4 (a) $n_1 = n_2 = n_3 = n_4 = 2, \alpha_p = 0.50, \alpha_f = 0.05$

θ_{31}	Ω_2	Ω_4				
		0.00000	2.41421	6.46410	8.47214	10.47723
1.00	0.00000	0.0024	−0.0164	−0.0380	−0.00437	−0.0462
	2.41421	0.0001	0.0007	0.0007	0.0010	−0.0024
	4.44949	0.0000	0.0006	0.0007	0.0019	0.0019
	6.46410	0.0000	0.0002	0.0007	0.0009	0.0011
	8.47214	0.0000	0.0000	0.0002	0.0003	0.0003
	10.47723	0.0000	0.0000	0.0001	0.0002	0.0004
5.00	0.00000	−0.0107	−0.0232	−0.0168	−0.0118	−0.0079
	2.41421	−0.0009	−0.0050	−0.0062	−0.0051	−0.0039
	4.44949	−0.0001	−0.0009	−0.0021	−0.0021	−0.0018
	6.46410	0.0001	−0.0001	−0.0006	−0.0007	−0.0007
	8.47214	0.0000	0.0001	−0.0001	−0.0002	−0.0005
	10.47723	0.0000	0.0000	0.0000	0.0000	0.0002
8.00	0.00000	−0.0105	−0.0149	−0.0066	−0.0038	−0.0021
	2.41421	−0.0016	−0.0043	−0.0031	−0.0020	−0.0012
	4.44949	0.0002	−0.0012	−0.0013	−0.0010	−0.0007
	6.46410	0.0000	−0.0003	−0.0005	−0.0004	−0.0004
	8.47214	0.0000	−0.0001	−0.0002	−0.0002	−0.0003
	10.47723	0.0000	0.0000	0.0000	0.0000	0.0006

* Power values of Ali's sometimes pool test procedure utilised for comparison were taken from his thesis [1]. Power values of Ali were subtracted from those of proposed SPT.

Table 3 (b) $n_1 = n_2 = 10, n_3 = n_4 = 2, \alpha_p = 0.50, \alpha_f = 0.05$

θ_{31}	Ω_2	Ω_4				
		0.00000	2.41421	6.46410	8.47214	10.47723
1.00	0.00000	−0.0016	−0.0157	−0.0323	−0.0333	−0.0322
	5.74166	0.0000	−0.0006	−0.0027	−0.0034	−0.0091
	10.00000	0.0000	0.0000	−0.0001	−0.0004	−0.0011
	12.07107	0.0000	0.0001	−0.0001	0.0007	0.0116
5.00	0.00000	−0.0073	−0.0087	−0.0023	−0.0001	0.0015
	5.74166	−0.0007	−0.0014	−0.0006	−0.0018	−0.0081
	10.00000	0.0000	−0.0003	0.0000	0.0006	−0.0156
	12.07107	−0.0003	−0.0012	−0.0003	0.0015	0.0357
8.00	0.00000	−0.0056	−0.0038	−0.0006	−0.0004	−0.0007
	5.74166	−0.0007	−0.0007	−0.0002	−0.0018	−0.0079
	10.00000	−0.0001	−0.0002	−0.0001	0.0016	0.0032
	12.07107	0.0000	0.0001	−0.0003	0.0041	0.0530

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